MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

Problem Set No. 3 — 6.635 Advanced Electromagnetism — Issued: 071015M
Fall Term 2007 — Due: 071029M

Reading assignment: Sections 4.7, 4.7A, J. A. Kong, "*Electromagnetic Wave Theory*", EMW Publishing, 2005

Problem P3.1

Show that, by choosing the contour as shown in Fig. 1.

$$I = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} d\alpha \, \frac{e^{\alpha t}}{\sqrt{\alpha+1}} = \sqrt{\frac{1}{\pi t}} e^{-t}$$

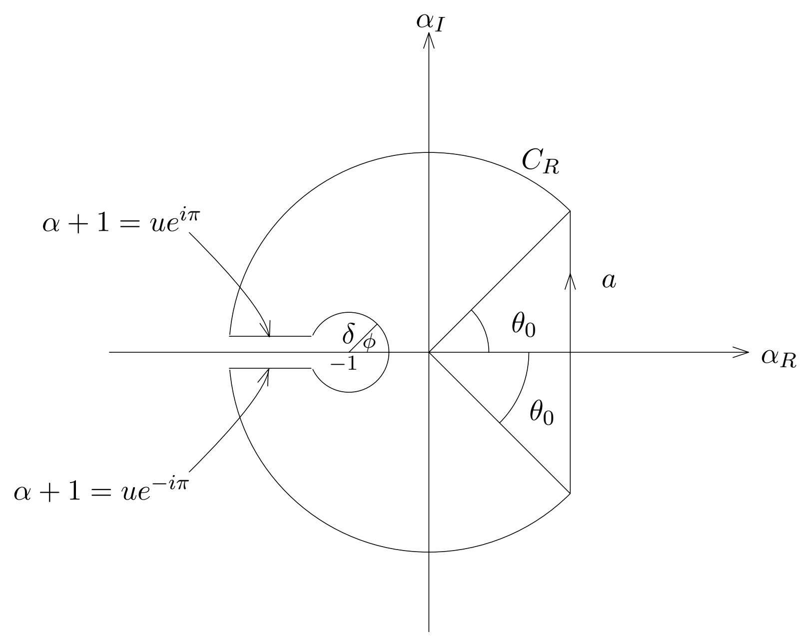

Fig. 1

Problem P3.2

Making use of the Cauchy-Riemann equations and Green's theorem

$$\oint_C d\bar{l} \cdot \overline{A} = \iint_D d\overline{s} \cdot (\nabla \times \overline{A})$$

prove Cauchy's Theorem, which states that if $f(\alpha)$ is analytic in a domain D and on the contour C bounding the domain D, then

$$\oint_C d\alpha \, f(\alpha) = 0$$

Problem P3.3

For a linear, temporally dispersive medium with conductivity σ, we have

$$\epsilon(\omega) = \epsilon_0 \left[1 + \frac{i\sigma}{\omega\epsilon_0} + \int_0^\infty d\tau\, \xi_e(\tau) e^{i\omega\tau}\right]$$

(a) Integrate $\oint_C d\alpha\, [\epsilon(\alpha) - \epsilon_\infty]/(\alpha - \omega)$ over a semicircle of infinite radius with the straight side along the real axis but indented around the points $\alpha = \omega$ and $\alpha = 0$. Show that the Kramers-Krönig relation is

$$\epsilon_R(\omega) - \epsilon_\infty = \frac{1}{\pi} \mathrm{PV} \int_{-\infty}^{\infty} dha\, \frac{\epsilon_I(\alpha)}{\alpha - \omega}$$

$$\epsilon_I(\omega) = -\frac{1}{\pi} \mathrm{PV} \int_{-\infty}^{\infty} a\alpha\, \frac{\epsilon_R(\alpha) - \epsilon_\infty}{\alpha - \omega} + \frac{\sigma}{\omega}$$

(b) Show that the result in (a) is independent of whether the indentation is made above or below the singularities at $\alpha = \omega$ and $\alpha = 0$.