

MASSACHUSETTS INSTITUTE OF TECHNOLOGY  
Department of Electrical Engineering and Computer Science

Problem Set No. 2  
Fall Term 2007

6.635 Advanced Electromagnetism

Issued: 071001M  
Due: 071015M

**Reading assignment:** Sections 7.1-7.3, J. A. Kong, “*Electromagnetic Wave Theory*”, EMW Publishing, 2005

**Problem P2.1**

- (a) What are the constitutive relations for a moving biisotropic medium which has the constitutive relations

$$\begin{bmatrix} cD' \\ H' \end{bmatrix} = \begin{bmatrix} p'\bar{I} & \ell'\bar{I} \\ -\ell'\bar{I} & q'\bar{I} \end{bmatrix} \begin{bmatrix} \bar{E}' \\ c\bar{B}' \end{bmatrix}.$$

in the rest frame of reference.

- (b) Find the constitutive relations of a biaxial medium moving along one of its principal axes.

**Problem P2.2**

Consider reflection and transmission by a moving uniaxial media with optic axis along the  $\hat{z}$  direction. Assume that the plane of incidence is parallel to the direction of motion.

- (a) Show that the dispersion relation is

$$k_x^2 + \frac{\mu_z}{\mu}(k_z - \omega\xi)^2 = \omega^2 \mu_z \epsilon$$

for TE waves, and

$$k_x^2 + \frac{\epsilon_z}{\epsilon}(k_z - \omega\xi)^2 = \omega^2 \mu \epsilon_z$$

for TM waves.

Note that the TE and TM waves correspond to the Type I and Type II waves discussed in Chapter 3.

- (b) Assume that the incident region 0 is also a moving medium. Total reflection occurs when the transmitted wave is evanescent. Find critical angles by requiring that  $k_{tx}^2 \leq 0$ . The condition for total reflection gives

$$(\omega - \chi_t k_z)^2 - \kappa_t \nu_t k_z^2 \leq 0$$

Consider moving isotropic media and approximate  $\kappa\nu \approx c^2/n^2$ ,  $\kappa_t\nu_t \approx c^2/n_t^2$ ,  $\chi \approx c\beta(1 - 1/n^2)$ , and  $\chi_t \approx c\beta_t(1 - 1/n_t^2)$  keeping only terms of the first order in  $\beta$ . For non-relativistic velocities,  $\omega = \chi k_z + ck/n$ . Show that

$$\sin \theta \geq \frac{n_t/n}{1 + n_t\beta_t(1 - 1/n_t^2) - n_t\beta(1 - 1/n^2)} \approx n_t\beta(1 - 1/n^2)$$

The last two terms are due to the motion of the media in regions 0 and  $t$ . The faster the medium in region  $t$  moves, the smaller the critical angle. Motion of the medium in region 0 induces opposite effects.