MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

Problem Set No. 1	6.635 Advanced Electromagnetism	Issued: 070910M
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Reading assignment: Chapter 7, J. A. Kong, "*Electromagnetic Wave Theory*", EMW Publishing, 2005

Problem P1.1

In a certain reference frame, a static uniform electric field E_0 is parallel to the z axis and a static uniform magnetic field $cB_0 = 2E_0$ forms a 30° angle with respect to $\hat{z}$. Determine the relative velocity of a reference frame in which the electric and magnetic fields are parallel.

Problem P1.2

An observer S observes a uniform electric field in the $\hat{x}$ direction, $\overline{E} = \hat{x}E_0$, and a uniform magnetic field in the $\hat{y}$ direction, $\overline{B} = \hat{y}B_0$. Let $E_0 > cB_0$. Find an observer S' moving relative to S with velocity v in the $\hat{z}$ direction, so that he observes only an electric field. Determine the electric field strength and the velocity v. Can you find an observer moving with a velocity less than c who observes only a magnetic field?

Problem P1.3

In the early stages of special relativity, the sudden disappearance of an absolute time scalar led to the well-known "twin paradox." The paradox as stated was that one of a pair of twins left home, traveled at a uniform (high) speed in some direction for a certain period of time, and then returned home to find himself younger than his brother. By the symmetry argument that motion is relative, it was argued that neither twin should have grown older than the other, and the validity of special relativity was challenged. In the following discussion we show that both of the twins agree that one is older than the other and the problem is not symmetric.

Let both A and B be at the origin in frame S; B starts to move at $t = 0$ with speed v in the positive $\hat{z}$ direction of S. As A reads time t, B moves back with speed v. Consider the following events:

Event 1: Twin B is at $z = vt$ when A reads time t. In frame S, this event is described by (ct, vt) where ct is the time coordinate of the event with the dimension length, and vt is the space coordinate of the event.

Event 2: As A reads time $2t$, both A and B are at $z = 0$. In frame S, this event is described by $(2ct, 0)$.

Consider two other frames of reference, S' and S''. Frame S' moves with velocity v in the positive direction of the z axis. Show that the space–time coordinates for the two events in frames S' and S'' are as listed in Table P7.1.3.

(a) Using the space–time coordinates of the two events, show that, by time dilation, twin A agrees that the total proper time interval for twin B is $2t/\gamma$, while his own coordinate time interval is $2t$.

Event	Observer		
	S	S'	S''
1	$ct(1, \beta)$	$ct(1/\gamma, 0)$	$ct[\gamma(1+\beta^2), 2\gamma\beta]$
2	$ct(2, 0)$	$ct(2\gamma, -2\gamma\beta)$	$ct(2\gamma, 2\gamma\beta)$

Table P7.1.3 Time–space coordinates of the two events in the three frames of reference. The first part in parentheses denotes time coordinates multiplied by c, and the second part denotes space coordinates.

(b) During the initial period before turning around, B is in S', and the elapsed time according to B is t/γ. Show that, according to B, the elapsed time during the final period after turning around is also t/γ. Thus twin B agrees with twin A that his time space is $2t/\gamma$, while that of A is $2t$.

(c) Show that observers moving uniformly relative to A, especially those in frame S' and S'', conclude that the proper elapsed time of B is less than that of A by a factor of $1/\gamma$.

(d) Suppose that twin B started his journey right after his birth and travelled with a speed $v = 0.8c$. If he comes back at 30 years of age, how old is his twin A?

(e) The problem is inherently asymmetrical; one twin has to turn, and it is this twin that experiences less proper time, $2t/\gamma$. If B does not turn, then, as A reads proper time $2t$, the coordinate time reading for B at two different locations, $z = 0$ and $z = 2vt$, is $2\gamma t$ because of the dilation of time. After turning around and meeting A again at $z = 0$, the proper time reading of B has been shown to be $2t/\gamma$. The effect of turning around causes a time difference of $2t(\gamma - 1/\gamma) = 2\gamma\beta^2 t$. Show that this "lost time" is equal to the time coordinate difference between S' and S'' for Event 1 in Table P7.1.3.

(f) It is interesting to imagine how B experiences the period of losing time during his turning around. Consider a third event, occurring when A reads time t at $z = 0$ in S. Find the time–space coordinates for Event 3 in S' and S''. Show that, according to S, Events 1 and 3 are simultaneous; according to S', Event 1 is earlier than Event 3; and according to S'', Event 3 is earlier than Event 1. At the turning time, twin B changes his frame from S' to S''. Show that B loses track of anything that happens at $z = 0$ during a time period $2\gamma\beta^2 t$.

Problem P1.4

The star Alpha Centauri is 4.3 light-years from Earth. Observer B leaves Earth in a rocket ship that travels toward this star at acceleration g. Halfway (2.15 light-years from Earth) from α Cen, B turns off the forward acceleration and accelerates backward toward Earth at g, so that the rocket arrives at α Cen with zero speed and turns back. On the return trip, at the halfway point, B again changes the direction of acceleration. Observer B arrives at Earth with zero speed. Show that B takes 7 years to complete the round trip, while the elapsed time on Earth is 12 years.