6.632 Solution to Problem Set 9

Solution P9.1

$$\overline{H}(\overline{r}) = (\hat{\theta}\cos\phi - \hat{\phi}\cos\theta\sin\phi)\frac{ik}{\pi r k_x k_y} e^{ikr}\sin\frac{k_x D_x}{2}\sin\frac{k_y D_y}{2}\,.$$

Solution P9.2

The fisrt zero of $\sin(kxw/2z)$ occurs at $kxw/2z = \pi$ which yields $2\pi w\theta/2\lambda = \pi$. Consequently $\theta = \lambda/w$.

For the Nth side lobe, $kxw/2z = kw\theta/2 = N\pi$, which yields $\theta_N = N\lambda/w$. Based on the assumption $\theta \approx x/z < 1$, we thus deduce that $N < w/\lambda$.

Solution P9.3

(a) For a biaxial medium,

$$\overline{\overline{\epsilon}} = \begin{bmatrix} \epsilon_x & 0 & 0 \\ 0 & \epsilon_y & 0 \\ 0 & 0 & \epsilon_z \end{bmatrix}$$

Since $\overline{\overline{\epsilon}}^t = \overline{\overline{\epsilon}}$, therefore the medium is reciprocal.

(b) For a moving biaxial medium,

$$\overline{\overline{C}}_{EB} = \gamma^2 \cdot \begin{bmatrix} c\epsilon'_x - \dfrac{\beta^2}{c\mu'} & 0 & 0 & 0 & \beta\left(-c\epsilon'_x + \frac{1}{c\mu'}\right) & 0 \\ 0 & c\epsilon'_y - \frac{\beta^2}{c\mu'} & 0 & \beta\left(c\epsilon'_y - \frac{1}{c\mu'}\right) & 0 & 0 \\ 0 & 0 & \frac{c\epsilon'_z}{\gamma^2} & 0 & 0 & 0 \\ 0 & \beta\left(-c\epsilon'_y + \frac{1}{c\mu'}\right) & 0 & -c\epsilon'_y\beta^2 + \frac{1}{c\mu'} & 0 & 0 \\ \beta\left(c\epsilon'_x - \dfrac{1}{c\mu'}\right) & 0 & 0 & 0 & -\beta^2 c\epsilon'_x + \frac{1}{c\mu'} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{\gamma^2 c\mu'} \end{bmatrix} \tag{1}$$

Since

$$\begin{bmatrix} c\overline{D} \\ \overline{H} \end{bmatrix} = \begin{bmatrix} \overline{\overline{P}} & \overline{\overline{L}} \\ \overline{\overline{M}} & \overline{\overline{Q}} \end{bmatrix} \begin{bmatrix} \overline{E} \\ c\overline{B} \end{bmatrix}$$

$$\overline{\overline{C}}_{EH} = \begin{bmatrix} \overline{\overline{\epsilon}} & \overline{\overline{\xi}} \\ \overline{\overline{\varsigma}} & \overline{\overline{\mu}} \end{bmatrix} = \frac{1}{c}\begin{bmatrix} \overline{\overline{P}} - \overline{\overline{L}}\cdot\overline{\overline{Q}}^{-1}\cdot\overline{\overline{M}} & \overline{\overline{L}}\cdot\overline{\overline{Q}}^{-1} \\ -\overline{\overline{Q}}^{-1}\cdot\overline{\overline{M}} & \overline{\overline{Q}}^{-1} \end{bmatrix} \tag{2}$$

We observe that $\overline{\overline{\xi}}$ and $\overline{\overline{\varsigma}}$ are not imaginary since there are no imaginary elements in (1). Thus the medium is nonreciprocal. The complementary medium is characterized by

$$\overline{\overline{\mu}}^c = \overline{\overline{\mu}}^t; \quad \overline{\overline{\epsilon}}^c = \overline{\overline{\epsilon}}^t;$$
$$\overline{\overline{\xi}}^c = -\overline{\overline{\varsigma}}^t; \quad \overline{\overline{\varsigma}}^c = -\overline{\overline{\xi}}^t.$$

(c) A biisotropic medium with imaginary χ is reciprocal.

(d) For a biisotropic medium with a real χ.

$$\overline{D} = \epsilon\overline{E} + \xi\overline{H}$$
$$\overline{B} = -\xi\overline{E} + \mu\overline{H}$$

the medium is reciprocal but lossy.
(e) For a ferrite in a dc magnetic field:

$$\overline{\overline{\mu}} = \begin{bmatrix} \mu_1 & -i\mu_2 & 0 \\ i\mu_2 & \mu_1 & 0 \\ 0 & 0 & \mu_0 \end{bmatrix}$$

and the medium is not reciprocal. The complementary medium is

$$\overline{\overline{\mu}}^c = \overline{\overline{\mu}}^t = \begin{bmatrix} \mu_1 & i\mu_2 & 0 \\ -i\mu_2 & \mu_1 & 0 \\ 0 & 0 & \mu_0 \end{bmatrix}$$