MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

Problem Set No. 9 — 6.632 Electromagnetic Wave Theory — Issued: 04/24/2008 R
Spring Term 2008 — Due: 05/01/2008 R

Reading assignment: Section 5.1, 5.2 ; J. A. Kong, "*Electromagnetic Wave Theory*," EMW Publishing, 2005.
Note: Quiz 2 will be held on May 8, 3-5pm at 26-314.

Problem P9.1

Determine the magnetic field $\overline{H}$ for the diffraction pattern in the far field produced by a plane wave with electric field $\overline{E} = \hat{y}E_0 e^{ikz}$ normally incident on a rectangular slit.

Problem P9.2

For diffraction by a slit of width $w = 2\ell$, the Fraunhofer field is found under the assumption that $dw \gg 1$ and $w/z \ll 1$ with the diffraction pattern proportional to $\sin(kx\ell/z)/(kx\ell/z)$ which peaks at $x = 0$. Let $x/z \approx \theta$ be the angular spread. Show that for the main beam angular spread, $\theta_1 \approx \lambda/2\ell$ where θ_1 is defined to be the first zero of the diffraction pattern. Show also that the maximum number of side lobes $N \leq w/\lambda$.

Problem P9.3

Which of the following media are lossless? Which of the following media are reciprocal? For the nonreciprocal ones, what are their complementary media?
(a) A biaxial medium with real constitutive parameters.
(b) A moving biaxial medium with

$$\begin{bmatrix} \overline{\overline{\epsilon}} & \overline{\overline{\xi}} \\ \overline{\overline{\varsigma}} & \overline{\overline{\mu}} \end{bmatrix} = \frac{1}{c}\begin{bmatrix} \overline{\overline{P}} - \overline{\overline{L}}\cdot\overline{\overline{Q}}^{-1}\cdot\overline{\overline{M}} & \overline{\overline{L}}\cdot\overline{\overline{Q}}^{-1} \\ -\overline{\overline{Q}}^{-1}\cdot\overline{\overline{M}} & \overline{\overline{Q}}^{-1} \end{bmatrix}$$

where

$$\begin{bmatrix} \overline{\overline{P}} & \overline{\overline{L}} \\ \overline{\overline{M}} & \overline{\overline{Q}} \end{bmatrix} = \gamma^2 \cdot \begin{bmatrix} c\epsilon'_x - \frac{\beta^2}{c\mu'} & 0 & 0 & 0 & \beta\left(-c\epsilon'_x + \frac{1}{c\mu'}\right) & 0 \\ 0 & c\epsilon'_y - \frac{\beta^2}{c\mu'} & 0 & \beta\left(c\epsilon'_y - \frac{1}{c\mu'}\right) & 0 & 0 \\ 0 & 0 & \frac{c\epsilon'_z}{\gamma^2} & 0 & 0 & 0 \\ 0 & \beta\left(-c\epsilon'_y + \frac{1}{c\mu'}\right) & 0 & -c\epsilon'_y\beta^2 + \frac{1}{c\mu'} & 0 & 0 \\ \beta\left(c\epsilon'_x - \frac{1}{c\mu'}\right) & 0 & 0 & 0 & -\beta^2 c\epsilon'_x + \frac{1}{c\mu'} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{\gamma^2 c\mu'} \end{bmatrix}$$

given $\beta = v/c$ and $\gamma = 1/\sqrt{1-\beta^2}$.

(c) A chiral medium

$$\overline{D} = \epsilon\overline{E} + i\chi\overline{H}$$
$$\overline{B} = \mu\overline{H} - i\chi\overline{E}$$

(d) A biisotropic medium with a real χ.

$$\overline{D} = \epsilon\overline{E} + \chi\overline{H}$$
$$\overline{B} = \mu\overline{H} - \chi\overline{E}$$

(e) A ferrite in a dc magnetic field with

$$\overline{\overline{\mu}} = \begin{bmatrix} \mu_1 & -i\mu_2 & 0 \\ i\mu_2 & \mu_1 & 0 \\ 0 & 0 & \mu_0 \end{bmatrix}$$